

# Design of Algorithmic Array Processors and its Applications

By S. Peng

Faculty of Computer and Information Sciences

Hosei University

Tokyo, Japan

# Introduction

- Design of **algorithm-oriented array processors** has been a popular research topic for more than two decades.
- The famous paper “**Systolic arrays (for VLSI)**” by H.T. Kung and C.E. Leiserson in 1978, and VLSI technology opened the era for the research on this area.
- The book “**VLSI Array Processors**” by S.Y. Kung provides major results on this area up to 1988.

# Introduction (Continue)

- In 1990s, the research on the area was extended to more advanced issues such as systematic designing tools, asynchronous arrays, programmable arrays, reconfigurable arrays, and fault-tolerance mechanisms etc., and became one of the fruitful branches in parallel and distributed systems.

# Introduction (Continue)

- Algorithmic array processors may derive a maximal concurrency by using **pipelining** and **parallel** processing. The key question is: How to map application algorithms onto array structures such that the **inherent concurrency** is fully achieved?
- In this talk, we will focus on this key issue and provide two **systematic methods** for designing efficient array processors.

# Array Processors

- The term “array processors” refers to a class of **special-purpose parallel computers** (see the book “Computer Architectures and Parallel Processing” by K. Hwang and F. Briggs in 1984)
- The term “algorithmic array processors” in this talk refers to a set of simple, locally interconnected processor elements (PEs). The PE’s **functionality** and the **communication structures** among PEs depend on the algorithm.

# Levels of Design

- For the design of computer systems, there are at least five levels of designs: algorithmic level, architectural level, logic level, circuit level and layout level.
- In this talk, we will focus on the levels 1 – 3 of design.
- In order to describe the design accurately, we need to define some terminologies.

# Terminologies

- **Algorithm expression** is a basic tool for a proper description of an algorithm for parallel and pipeline processing.
- A **single assignment code** is a form where every variable is assigned one value only during the execution of the algorithm.
- **Recursive algorithms** are those algorithms expressed by recursive equations with **space-time indices** (one index for time and the other indices for space).

# Terminologies (Continue)

- A **dependency graph** (DG) is a graph that shows the dependence of the computations that occur in an algorithm. A DG is the **graphical representation** of a single assignment algorithm.
- An algorithm is **computable** if and only if its DG contains no cycles.
- A **locally recursive algorithm** is an algorithm whose DG has only local dependencies.
- A DG is **shift-invariant** if the dependence arcs in the index space are independent of their positions.

# Mapping Algorithm to DG

- Although many methods have been proposed to **construct a DG from sequential code**, a formal and automatic methodology remains a major open research problem.
- Finding alternative ways for the **ordering of the single assignment code** of an algorithm such that the corresponding DGs are suitable for constructing efficient array processors is the key for successful design of array processors.

# Re-indexing of DG

- A useful technique for modifying the DG is by the **re-indexing** scheme. For example, when there is no **permissible linear schedule** or systolic schedule for the original DG, it is often desirable to modify the DG so that such a desired schedule may be obtained.
- In this talk, we will show how to find a re-indexing scheme and construct optimal array processors for **division-free Gaussian elimination** algorithm using a **top-down** approach.

# A Top-down Design Method: Projection of DG

- In a **linear projection**, nodes of the DG in a certain straight line are projected to a PE in the processor array along a projection direction.
- To guarantee that each PE executes at most one computation, there are **conditions** for a admissible projections.
- We use a software tool (**S4CAD**) for creating all admissible array processors of a given DG automatically.
- This method is suitable for a locally recursive algorithm with **2-D** or **3-D** DG.

# The Top-down Design Method (Continue)

- The design begins with investigating the DG of a locally recursive algorithm, and then uses re-indexing techniques if necessary to re-organize the DG for **the higher degree of concurrence**.
- The second part of the design is done through a software tool **S4CAD** (System of Systolic Structures Synthesis). The S4CAD automatically produces all admissible array processors, among which the designer can choose the **optimal** one.
- The website of S4CAD:  
<http://gemini.u-aizu.ac.jp/HPCC/S4CAD>

# A Bottom-up Design Method for Multi-dimensional Arrays

- For many **multi-dimensional** signal/image transforms (e.g., DFT, WHT, DCT), the kernels are **separable**. That is, a  **$k$ -D** transform can be done by performing consecutive **1-D** transforms  **$k$**  times, each along **distinct** dimension.
- For such applications, we proposed a **bottom-up** design method using different types of 1-D array processors as building blocks.

# The Idea

- Three types of linear array processors with **distinct I/O format** are created as building blocks:
  - ◆ **Type I**: Input data are fed into the array in a pipelined fashion and output data are generated and stored inside each PE.
  - ◆ **Type II**: Input data are from local registers and output data are delivered to the next PE.
  - ◆ **Type III**: Both input and output data are fed/stored in the local registers.

# The Method

- **Step 1:** Find a **2-D localized DG** of the 1-D signal/image transform.
- **Step 2:** Find the three types of linear array processors through **projection** and **pipelining** techniques applying on the 2-D DG.
- **Step 3:** Refine the design to **reduce I/O overhead** through analyzing the kernel of the transform.
- **Step 4:** Build an  **$k$ -D array processor ( $k = 2$  or  $3$ )** for the  **$k$ -D transform** as follows: The first and the last dimensions use **type I** and **type II** structures, and the intermediate dimension uses **type III** structure.

# An Array Processor for Video Applications

- The bottom-up design method is also called the **dimensional-splitting method** which can be used to construct array processors for multi-dimensional image/signal transforms with separable kernel.
- The construction of the **efficient linear array processors** is the key issue for this approach.
- We will show, as an example, how to design array processors for **2-D** or **3-D DCT** which is practically useful for many video applications.

# Concluding Remarks

- Because of changing technologies and applications in the last few years, there are many interesting research topics on this area. We list some possible directions of research below:
  - ◆ Design array processors for the new application algorithms: **computer vision, 3-D computer graphics, multimedia, virtual and cyber worlds.**
  - ◆ Design **automatic design/analysis software tools** (e.g., using java graphical interfaces) for array processors.
  - ◆ Investigate array processor designs that are created by the proposed methods in **circuit or layout** levels.